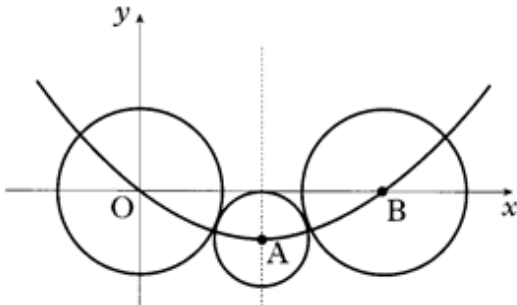
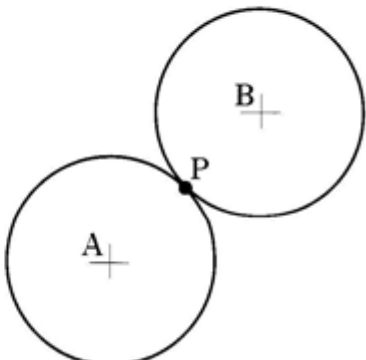


Circle Equation

Basic/General Equation

<p>2003 P1</p>	<p>11.</p> <ul style="list-style-type: none"> • O, A and B are the centres of the three circles shown in the diagram below. • The two outer circles are congruent and each touches the smallest circle. • Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$. • The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.  <p>(a) (i) State the coordinates of A and find the length of the line OA. 2</p> <p>(ii) Hence find the equation of the circle with centre B. 3</p> <p>(b) The equation of the parabola can be written in the form $y = px(x + q)$. Find the values of p and q. 2</p>
<p>2005 P1</p>	<p>2. Two congruent circles, with centres A and B, touch at P.</p> <p>Relative to suitable axes, their equations are</p> $x^2 + y^2 + 6x + 4y - 12 = 0 \text{ and } x^2 + y^2 - 6x - 12y + 20 = 0.$ <p>(a) Find the coordinates of P. 3</p> <p>(b) Find the length of AB. 2</p> 
<p>2006 P2</p>	<p>4. The circles with equations $(x - 3)^2 + (y - 4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre.</p> <p>Determine the radius of the larger circle. 5</p>

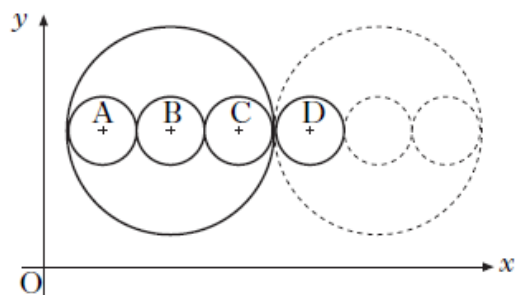
2007
P1

5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x -axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.

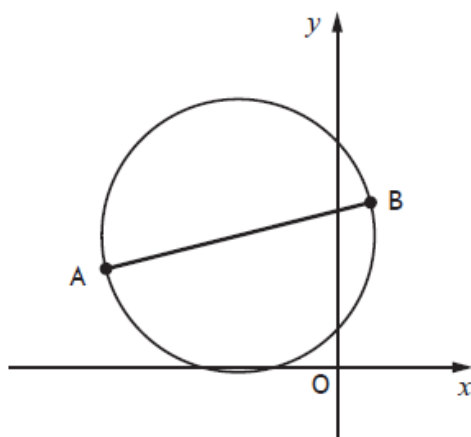


5

2016
P1

4. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.



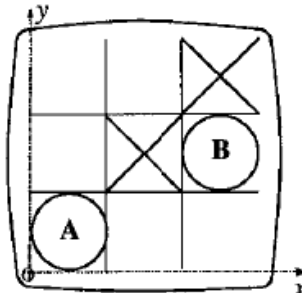
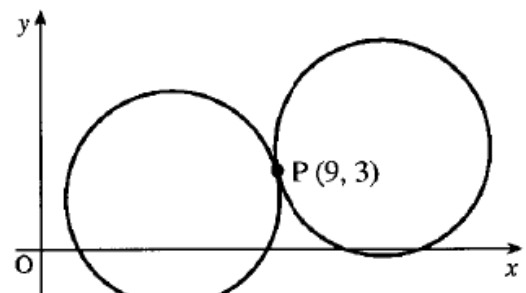

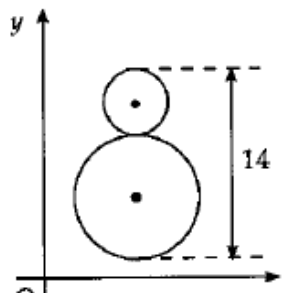
Find the equation of this circle.

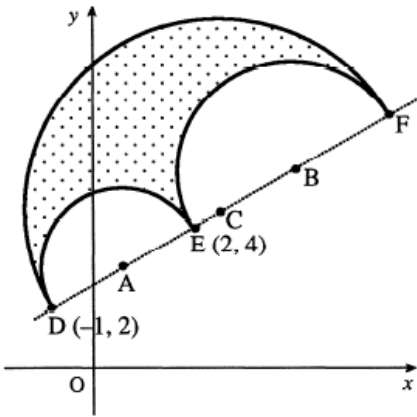
3

2017 P2	<div data-bbox="220 98 1465 136"> <p>10. (a) Show that the points $A(-7, -2)$, $B(2, 1)$ and $C(17, 6)$ are collinear. 3</p> </div> <div data-bbox="288 185 1342 248"> <p>Three circles with centres A, B and C are drawn inside a circle with centre D as shown.</p> </div> <div data-bbox="501 309 1053 860"> </div> <div data-bbox="288 913 1155 952"> <p>The circles with centres A, B and C have radii r_A, r_B and r_C respectively.</p> </div> <div data-bbox="288 965 461 1120"> <ul style="list-style-type: none"> • $r_A = \sqrt{10}$ • $r_B = 2r_A$ • $r_C = r_A + r_B$ </div> <div data-bbox="288 1149 1465 1184"> <p>(b) Determine the equation of the circle with centre D. 4</p> </div>
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Pre 2000 - Basic/General Equation

1	<div data-bbox="186 1335 735 1408"> <p>Find the equation of the circle with centre $(-3, 4)$ and passing through the origin.</p> </div> <div data-bbox="847 1330 1267 1738"> </div> <div data-bbox="1409 1368 1425 1395"> <p>2</p> </div>
2	<div data-bbox="186 1776 1230 1809"> <p>Find the equation of the circle which has $P(-2, -1)$ and $Q(4, 5)$ as the end points of a diameter.</p> </div> <div data-bbox="1409 1776 1425 1803"> <p>3</p> </div>

3	<p>This diagram shows a computer-generated display of a game of noughts and crosses.</p> <p>Relative to the coordinate axes which have been added to the display, the "nought" at A is represented by a circle with equation $(x-2)^2 + (y-2)^2 = 4$.</p> <p>(a) Find the centre of the circle at B.</p> <p>(b) Find the equation of the circle at B.</p>	 <div style="text-align: right;">3 1</div>
4	<p>Two identical circles touch at the point P (9, 3) as shown in the diagram. One of the circles has equation $x^2 + y^2 - 10x - 4y + 12 = 0$.</p> <p>Find the equation of the other circle.</p>	 <div style="text-align: right;">5</div>
5	<p>An ancient Stone Circle has a processional pathway from the Heelstone to the centre of the Stone Circle. In the picture above, the Heelstone is on the left and the dotted line represents the processional pathway.</p> <p>With suitable axes and using the heelstone as the origin, the equation of the Stone Circle is $x^2 + y^2 - 8x - 6y + 21 = 0$.</p> <p>Given that 1 unit represents 15metres, calculate the distance in metres from the Heelstone to the nearest point on the edge of the Circle.</p>	 <div style="text-align: right;">5</div>
6	<p>A bakery firm makes gingerbread men each 14cm high with a circular "head" and "body".</p> <p>The equation of the "body" is $x^2 + y^2 - 10x - 12y + 45 = 0$ and the line of centres is parallel to the y-axis. Find the equation of the "head".</p>	 <div style="text-align: right;">5</div>

7	<p>The shape shown in the diagram is composed of 3 semicircles with centres A, B and C which lie on a straight line.</p> <p>DE is a diameter of one of the semicircles. The coordinates of D and E are $(-1, 2)$ and $(2, 4)$.</p> <p>(a) Find the equation of the circle with centre A and diameter DE. (3)</p> <p>The circle with centre B and diameter EF has equation $x^2 + y^2 - 16x - 16y + 76 = 0$.</p> <p>(b) (i) Write down the coordinates of B. (3) (ii) Determine the coordinates of F and C.</p> <p>(c) In the diagram the perimeter of the shape is represented by the thick black line. Show that the perimeter is $5\pi\sqrt{13}$ units. (3)</p>	
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Using $r = \sqrt{(g^2 + f^2 - c)}$

2000 P1	A6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle?	5
2014 P2	8. Given that the equation $x^2 + y^2 - 2px - 4py + 3p + 2 = 0$ represents a circle, determine the range of values of p .	5
2015 P1	14. The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points. What is the value of k ?	2

Pre 2000 - Using $r = \sqrt{(g^2 + f^2 - c)}$

1	Explain why the equation $x^2 + y^2 + 2x + 3y + 5 = 0$ does not represent a circle.	2
2	For what range of values of c does the equation $x^2 + y^2 - 6x + 4y + c = 0$ represent a circle?	3

Using Perpendicular Bisectors to find the centre

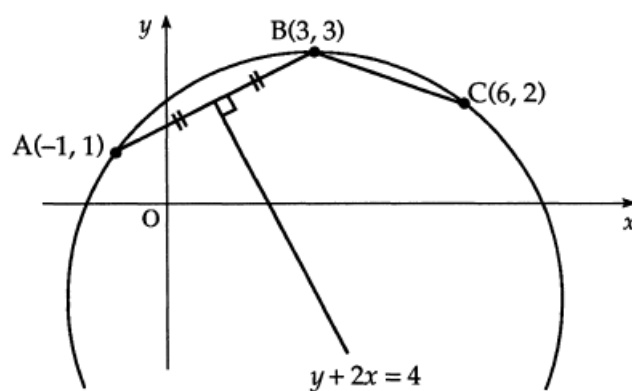
<p>2001 P2</p>	<p>7. Triangle ABC has vertices A(2, 2), B(12, 2) and C(8, 6).</p> <p>(a) Write down the equation of l_1, the perpendicular bisector of AB. 1</p> <p>(b) Find the equation of l_2, the perpendicular bisector of AC. 4</p> <p>(c) Find the point of intersection of lines l_1 and l_2. 1</p> <p>(d) Hence find the equation of the circle passing through A, B and C. 2</p> <div data-bbox="837 201 1396 537"> </div>
<p>2015 SP P2</p>	<p>5.</p> <div data-bbox="526 761 1005 1209"> </div> <p>Points A(−1, −1) and B(7, 3) lie on the circumference of a circle with centre C, as shown in the diagram.</p> <p>(a) Find the equation of the perpendicular bisector of AB. 4</p> <p>CB is parallel to the x-axis.</p> <p>(b) Find the equation of the circle, passing through A and B, with centre C. 4</p>

Pre 2000 - Using Perpendicular Bisectors to find the centre

1

- (a) In the diagram, A is the point $(-1, 1)$, B is $(3, 3)$ and C is $(6, 2)$. The perpendicular bisector of AB has equation $y + 2x = 4$. Find the equation of the perpendicular bisector of BC.

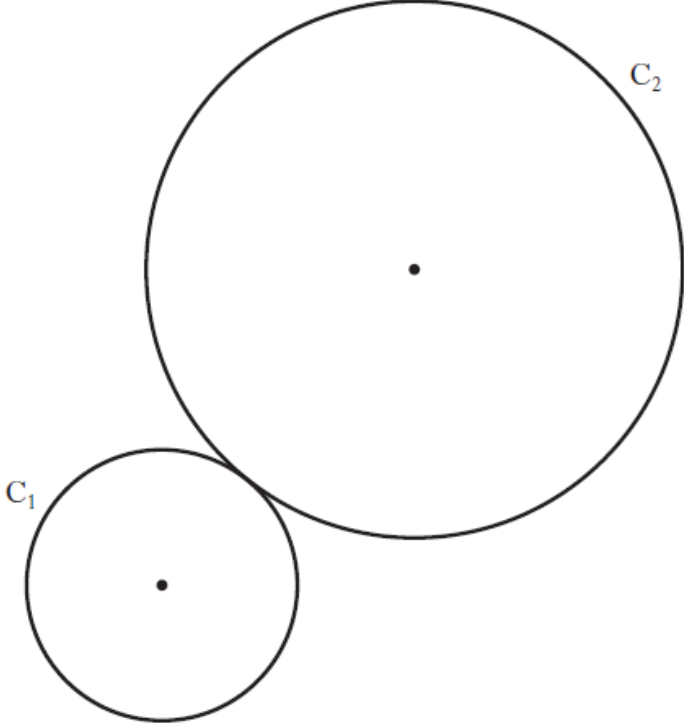
(4)

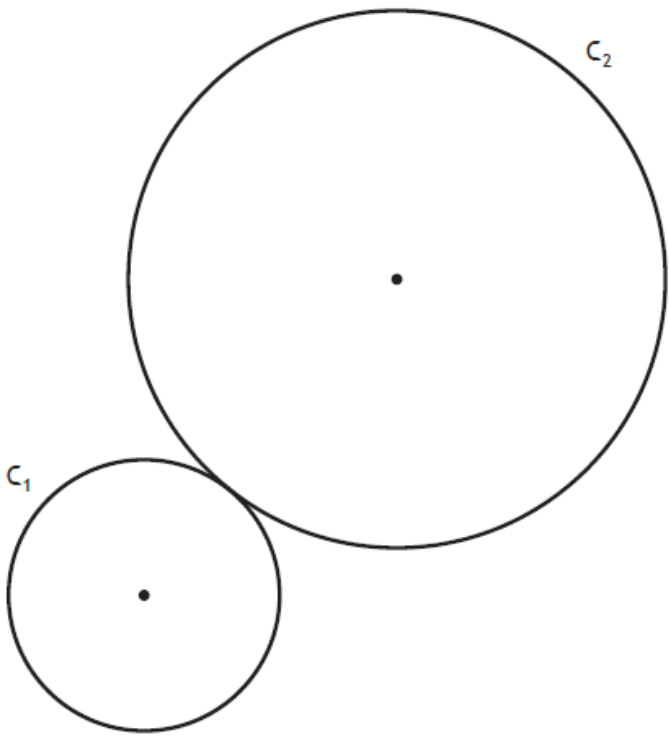


- (b) Find the centre and the equation of the circle which passes through A, B and C.

(6)

Touching Circles and Distance between Circles

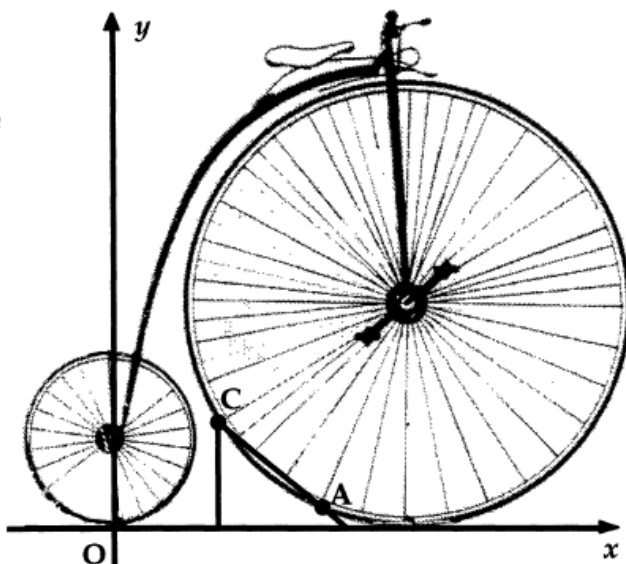
2011 P2	<p>7. Circle C_1 has equation $(x + 1)^2 + (y - 1)^2 = 121$. A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1. The circles have no points of contact. What is the range of values of p?</p> <p style="text-align: right;">9</p>
2015 OLD P2	<p>5. Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$. The centre of circle C_2 is $(9, 11)$. Circles C_1 and C_2 touch externally.</p>  <p>(a) Determine the radius of C_2.</p> <p style="text-align: right;">4</p> <p>A third circle, C_3, is drawn such that:</p> <ul style="list-style-type: none"> • both C_1 and C_2 touch C_3 internally • the centres of C_1, C_2 and C_3 are collinear. <p>(b) Determine the equation of C_3.</p> <p style="text-align: right;">4</p>

2015 P2	<p>5. Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$. The centre of circle C_2 is $(9, 11)$. Circles C_1 and C_2 touch externally.</p>  <p>(a) Determine the radius of C_2. 4</p> <p>A third circle, C_3, is drawn such that:</p> <ul style="list-style-type: none"> • both C_1 and C_2 touch C_3 internally • the centres of C_1, C_2 and C_3 are collinear. <p>(b) Determine the equation of C_3. 4</p>
2016 P2	<p>4. Circles C_1 and C_2 have equations $(x + 5)^2 + (y - 6)^2 = 9$ and $x^2 + y^2 - 6x - 16 = 0$ respectively.</p> <p>(a) Write down the centres and radii of C_1 and C_2. 4</p> <p>(b) Show that C_1 and C_2 do not intersect. 3</p>

1

A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.

With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is $x^2 + y^2 - 6y = 0$ and the equation of the front wheel is $x^2 + y^2 - 28x - 20y + 196 = 0$.



- (a) (i) Find the distance between the centres of the two wheels.
 (ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. (8)
- (b) B(7,3) is half-way between A and C, and P is the centre of the front wheel.
 (i) Find the gradient of PB.
 (ii) Hence find the equation of AC and the coordinates of A and C. (8)

Intersection of Lines and Circles

<p>2008 P2</p>	<p>4. (a) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. 2</p> <p>(b) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$. Find the distance between the centres of these two circles and hence show that the circles intersect. 4</p> <p>(c) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles. Find the coordinates of the points of intersection of the two circles. 5</p>
<p>2012 P2</p>	<p>2. (a) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.</p> <div data-bbox="740 698 1024 1055" data-label="Image"> </div> <p style="text-align: right;">Diagram 1</p> <p>Find the coordinates of P and Q. 6</p> <p>(b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.</p> <div data-bbox="657 1294 1011 1646" data-label="Image"> </div> <p style="text-align: right;">Diagram 2</p> <p>Determine the equation of this second circle. 6</p>
<p>2014 P1</p>	<p>23. (a) Find P and Q, the points of intersection of the line $y = 3x - 5$ and the circle C_1 with equation $x^2 + y^2 + 2x - 4y - 15 = 0$. 4</p> <p>(b) T is the centre of C_1. Show that PT and QT are perpendicular. 3</p> <p>(c) A second circle C_2 passes through P, Q and T. Find the equation of C_2. 3</p>

2. (a) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

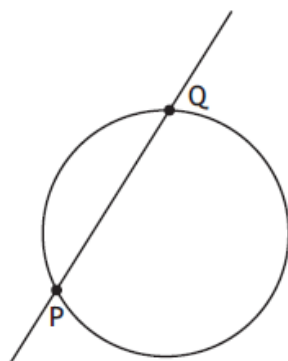


Diagram 1

Find the coordinates of P and Q.

6

- (b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

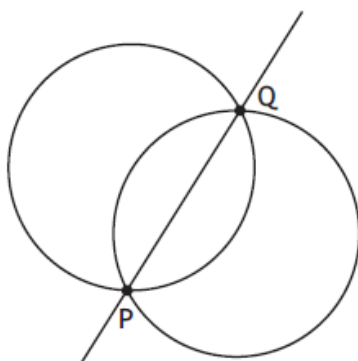
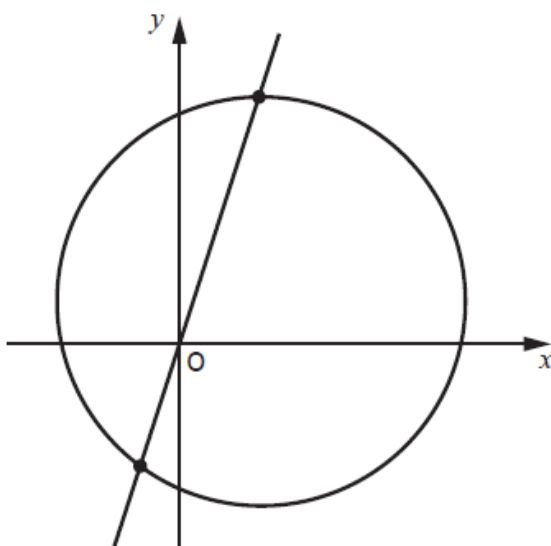


Diagram 2

Determine the equation of this second circle.

6

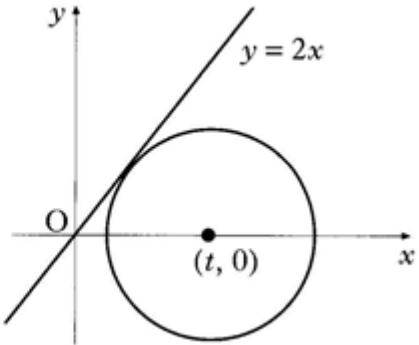
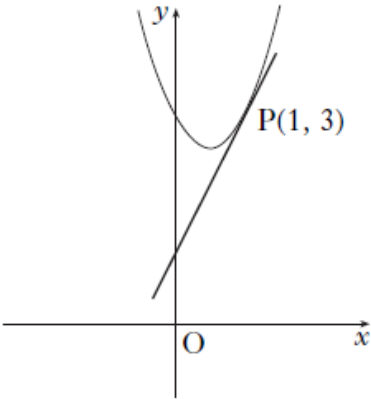
3. The line $y = 3x$ intersects the circle with equation $(x - 2)^2 + (y - 1)^2 = 25$.

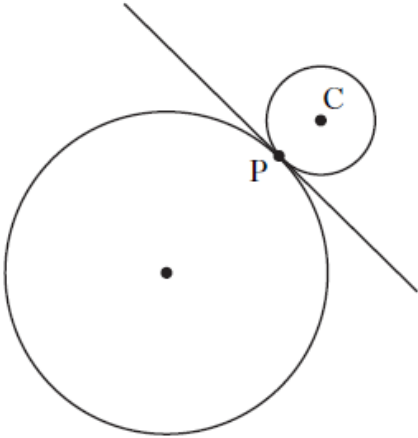


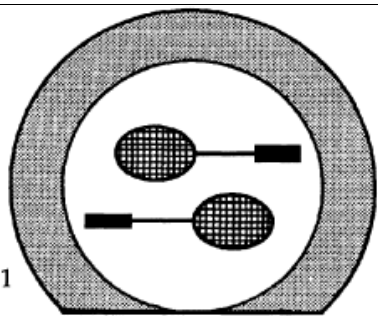
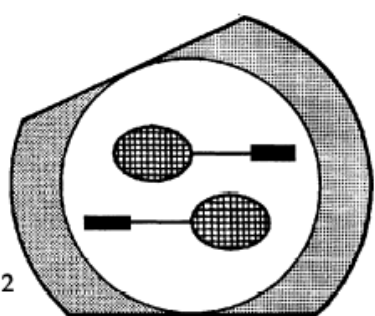
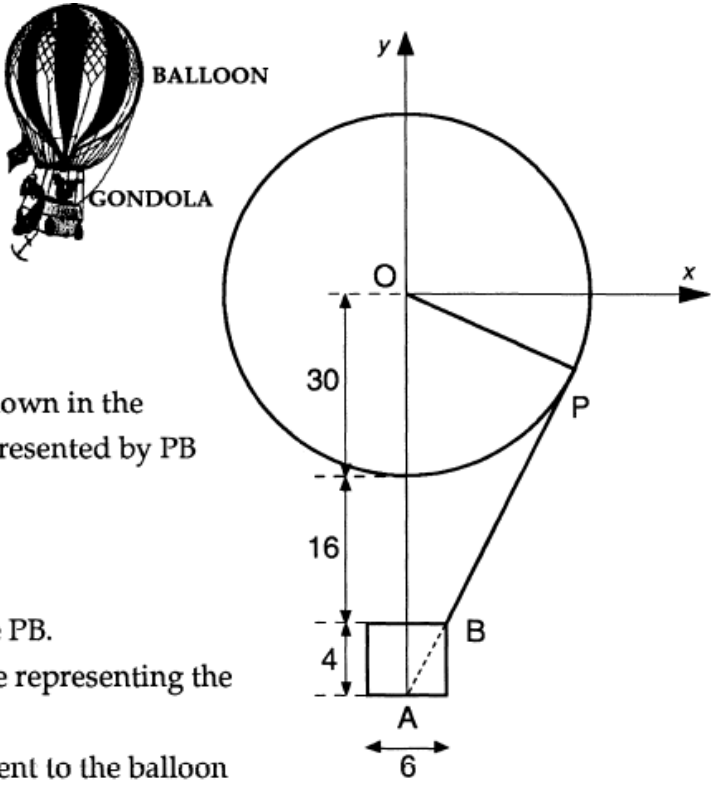
Find the coordinates of the points of intersection.

5

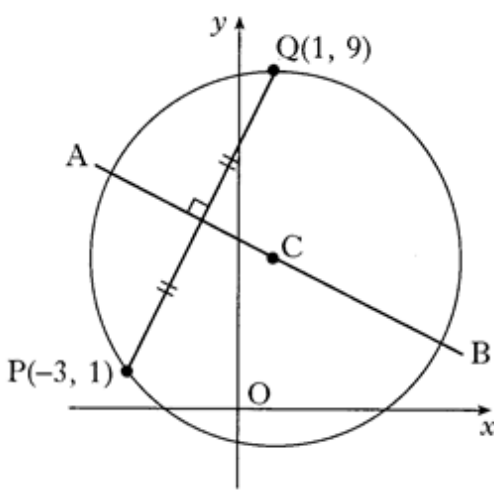
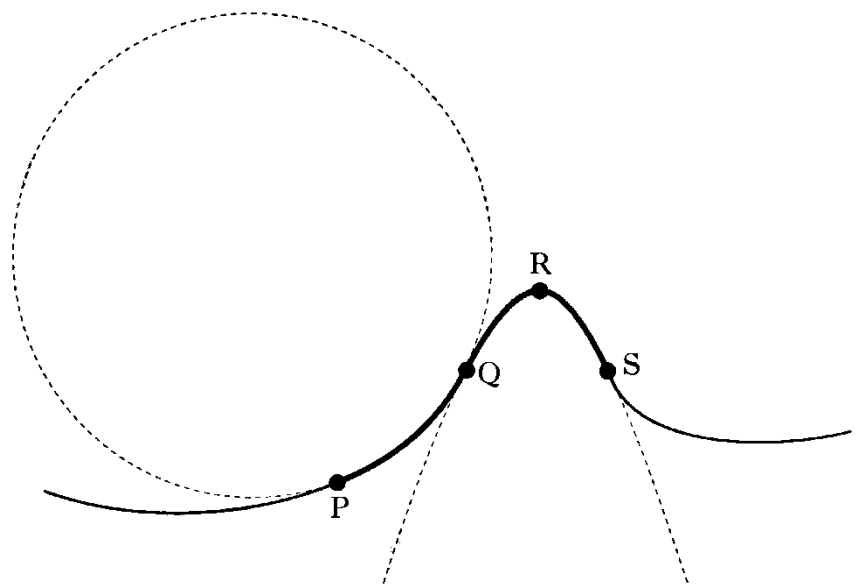
Tangency

5. (JAN) 02 P2	<p>The line $y + 2x = k$, $k > 0$, is a tangent to the circle $x^2 + y^2 - 2x - 4 = 0$.</p> <p>(a) Find the value of k. 7</p> <p>(b) Deduce the coordinates of the point of contact. 2</p>
2003 P2	<p>4. (a) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. 5</p> <p>(b) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. 6</p>
2005 P1	<p>11. (a) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units. Write down the equation of the circle. 1</p> <p>(b) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. 5</p> 
2007 P2	<p>3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. 6</p>
2008 SP1 P2	<p>4. The diagram shows a parabola with equation $y = 2x^2 - 2x + 3$. A tangent to the parabola has been drawn at $P(1, 3)$.</p> <p>(a) Find the equation of this tangent. 4</p> <p>A circle has equation $x^2 + y^2 + 8y + 11 = 0$.</p> <p>(b) Show that the line from (a) is also a tangent to this circle and state the coordinates of the point of contact Q. 6</p> <p>(c) Determine the ratio in which the y-axis cuts the line QP. 3</p> 

2010 P2	<p>3. (a) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.</p> <p>(ii) Find the coordinates of the point of contact, P.</p> <p>(b) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (a) and a second smaller circle with centre C.</p>  <p>The line $y = 3 - x$ is a common tangent at the point P.</p> <p>The radius of the larger circle is three times the radius of the smaller circle.</p> <p>Find the equation of the smaller circle.</p>	5
2016 P1	<p>8. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact.</p>	5

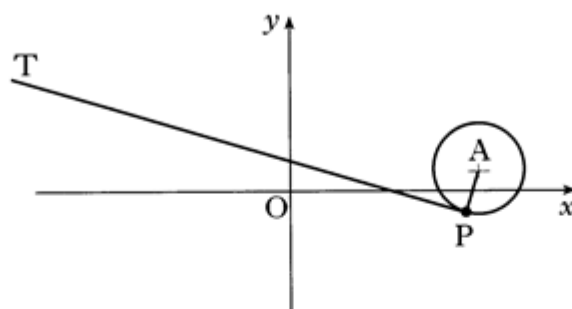
1	Find the possible values of k for which the line $x - y = k$ is a tangent to the circle $x^2 + y^2 = 18$.	5
2	The line $y = -1$ is a tangent to a circle which passes through $(0, 0)$ and $(6, 0)$. Find the equation of this circle.	6
3	<p>A sports club awards trophies in the form of paperweights bearing the club crest. Diagram 1 shows the front view of one of these paperweights. Each is made from two different types of glass. The two circles are concentric and the base line is a tangent to the inner circle.</p> <p>(a) Relative to x, y coordinate axes, the equation of the outer circle is $x^2 + y^2 - 8x + 2y - 19 = 0$ and the equation of the base line is $y = -6$. Show that the equation of the inner circle is $x^2 + y^2 - 8x + 2y - 8 = 0$. (4)</p> <p>(b) An alternative form of the paperweight is made by cutting off a piece of glass from the original design along a second line with equation $3x - 4y + 9 = 0$ as shown in diagram 2. Show that this line is a tangent to the inner circle and state the coordinates of the point of contact. (7)</p>	 <p>Diagram 1</p>  <p>Diagram 2</p>
4	<p>A spherical hot-air balloon has radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.</p> <p>Co-ordinate axes are chosen as shown in the diagram. One of the cables is represented by PB and PBA is a straight line.</p> <p>(a) Find the equation of the cable PB. (3)</p> <p>(b) State the equation of the circle representing the balloon. (1)</p> <p>(c) Prove that this cable is a tangent to the balloon and find the co-ordinates of the point P. (5)</p>	

Equations of Tangents

<p>2000 P2</p>	<p>A2. (a) Find the equation of AB, the perpendicular bisector of the line joining the points P(-3, 1) and Q(1, 9).</p> <p>(b) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle.</p> <p>(c) The tangents at P and Q intersect at T.</p> <p>Write down</p> <p>(i) the equation of the tangent at Q</p> <p>(ii) the coordinates of T.</p>	 <p>4</p> <p>3</p> <p>2</p>
<p>2002 P1</p>	<p>1. The point P(2, 3) lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P.</p>	<p>4</p>
<p>7. (JAN) 02 P1</p>	<p>The side view of part of a roller coaster ride is shown by the path PQRS. The curve PQ is an arc of the circle with equation $x^2 + y^2 + 4x - 10y + 9 = 0$. The curve QRS is part of the parabola with equation $y = -x^2 + 6x - 5$. The point Q has coordinates (2, 3).</p>  <p>(a) Find the equation of the tangent to the circle at Q.</p> <p>(b) Show that this tangent to the circle at Q is also the tangent to the parabola at Q.</p>	<p>4</p> <p>2</p>

2004
P2

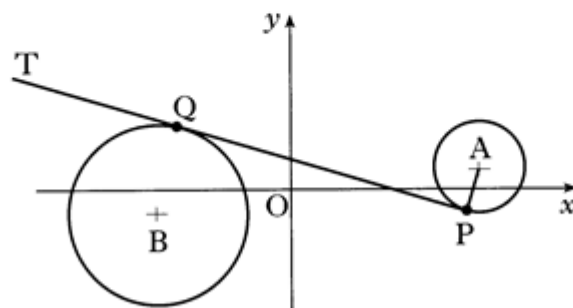
8. The circle with centre A has equation $x^2 + y^2 - 12x - 2y + 32 = 0$. The line PT is a tangent to this circle at the point P(5, -1).



- (a) Show that the equation of this tangent is $x + 2y = 3$.

4

The circle with centre B has equation $x^2 + y^2 + 10x + 2y + 6 = 0$.



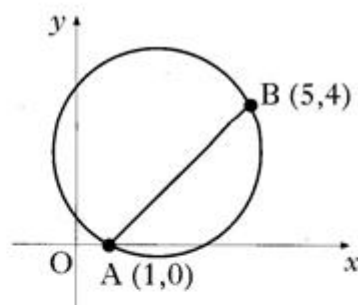
- (b) Show that PT is also a tangent to this circle.
(c) Q is the point of contact. Find the length of PQ.

5

2

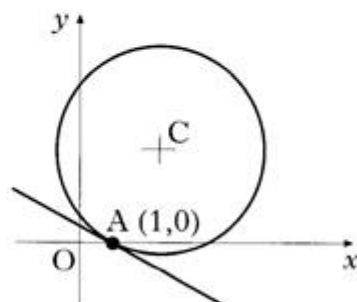
2005
P2

3. (a) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram. Show that the equation of the perpendicular bisector of chord AB is $x + y = 5$.



4

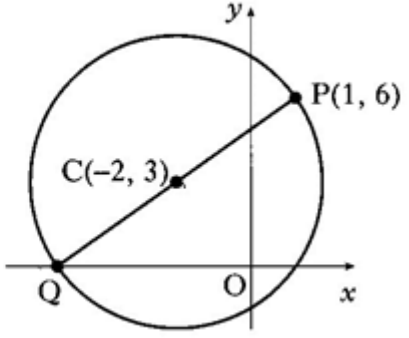
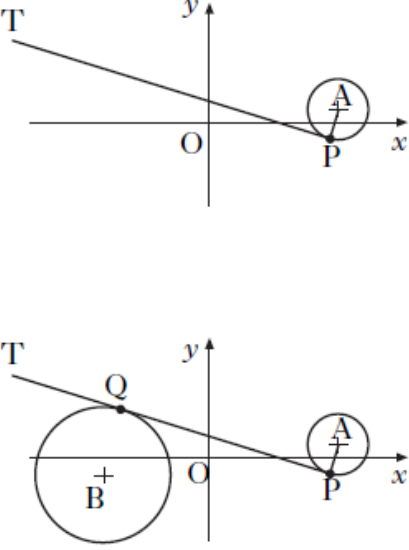
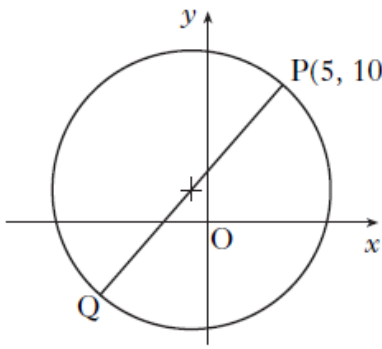
- (b) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1$. Find the equation of the radius CA.



4

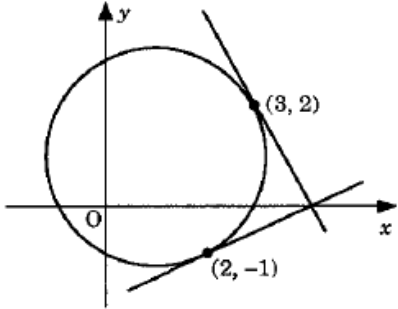
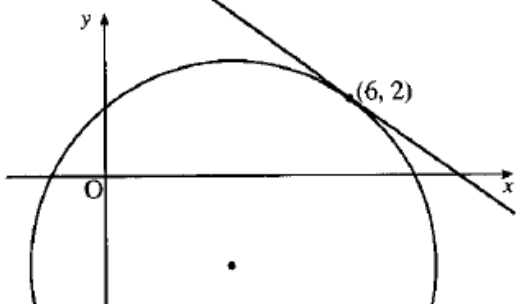
- (c) (i) Determine the coordinates of the point C.
(ii) Find the equation of the circle.

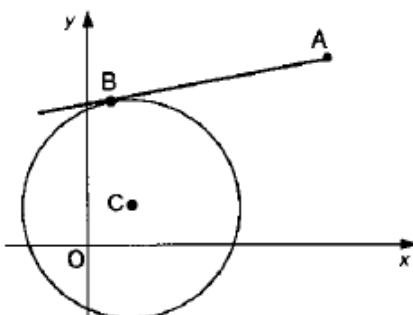
4

2006 P1	<p>2. A circle has centre $C(-2, 3)$ and passes through $P(1, 6)$.</p> <p>(a) Find the equation of the circle.</p> <p>(b) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q.</p>	 <div style="display: flex; justify-content: space-between; align-items: center;"> <div> 2 4 </div> </div>
2008 SP2 P2	<p>6. The circle with centre A has equation $x^2 + y^2 - 12x - 2y + 32 = 0$. The line PT is a tangent to this circle at the point $P(5, -1)$.</p> <p>(a) Show that the equation of this tangent is $x + 2y = 3$.</p> <p>The circle with centre B has equation $x^2 + y^2 + 10x + 2y + 6 = 0$.</p> <p>(b) Show that PT is also a tangent to this circle.</p> <p>(c) Q is the point of contact. Find the length of PQ.</p>	 <div style="display: flex; justify-content: space-between; align-items: center;"> <div> 4 5 2 </div> </div>
2009 P2	<p>4. (a) Show that the point $P(5, 10)$ lies on circle C_1 with equation $(x + 1)^2 + (y - 2)^2 = 100$.</p> <p>(b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q.</p> <p>(c) Two circles, C_2 and C_3, touch circle C_1 at Q. The radius of each of these circles is twice the radius of circle C_1. Find the equations of circles C_2 and C_3.</p>	 <div style="display: flex; justify-content: space-between; align-items: center;"> <div> 1 5 </div> </div> <div style="text-align: right; margin-top: 20px;">4</div>

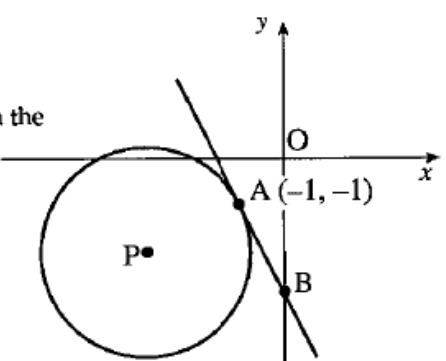
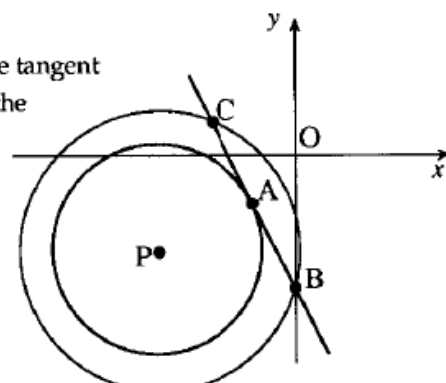
2013 P1	<p>22. A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.</p> <p>(a) Write down the centre and calculate the radius of C_1. 2</p> <p>(b) The point $P(3, 2)$ lies on the circle C_1. Find the equation of the tangent at P. 3</p> <p>(c) A second circle C_2 has centre $(10, -1)$. The radius of C_2 is half of the radius of C_1. Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$. 3</p> <p>(d) Show that the tangent found in part (b) is also a tangent to circle C_2. 4</p>
2015 P1	<p>11. $T(-2, -5)$ lies on the circumference of the circle with equation</p> $(x + 8)^2 + (y + 2)^2 = 45.$ <p>(a) Find the equation of the tangent to the circle passing through T. 4</p> <p>(b) This tangent is also a tangent to a parabola with equation $y = -2x^2 + px + 1 - p$, where $p > 3$. Determine the value of p. 6</p>
2017 P1	<p>2. The point $P(-2, 1)$ lies on the circle $x^2 + y^2 - 8x - 6y - 15 = 0$. Find the equation of the tangent to the circle at P. 4</p>

Pre 2000 - Equations of Tangents

1	<p>The circle shown in the diagram has equation $(x - 1)^2 + (y - 1)^2 = 5$. Tangents are drawn at the points $(3, 2)$ and $(2, -1)$. Write down the coordinates of the centre of the circle and hence show that the tangents are perpendicular to each other.</p>  4
2	<p>The circle shown has equation $(x - 3)^2 + (y + 2)^2 = 25$. Find the equation of the tangent at the point $(6, 2)$.</p>  4
3	<p>Find the equation of the tangent at the point $(3, 4)$ on the circle $x^2 + y^2 + 2x - 4y - 15 = 0$. 4</p>
4	<p>Find the equation of the tangent at the point $(3, 1)$ on the circle $x^2 + y^2 - 4x + 6y - 4 = 0$. 5</p>

5	<p>AB is a tangent at B to the circle with centre C and equation $(x-2)^2 + (y-2)^2 = 25$.</p> <p>The point A has co-ordinates (10, 8).</p> <p>Find the area of triangle ABC.</p>		5
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Mixed Questions

2001 P1	<p>11. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.</p> <p>(a) (i) Show that the radius of circle P is $4\sqrt{2}$.</p> <p>(ii) Hence show that circles P and Q touch.</p> <p>(b) Find the equation of the tangent to circle Q at the point $(-4, 1)$.</p> <p>(c) The tangent in (b) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$.</p>	<p>4</p> <p>3</p> <p>3</p>
1	<p>(a) The diagram shows a circle, centre P, with equation $x^2 + y^2 + 6x + 4y + 8 = 0$. Find the equation of the tangent at the point A $(-1, -1)$ on the circle.</p> <p>(b) The tangent crosses the y-axis at B. Find the coordinates of B.</p> <p>(c) Another circle, centre P, is drawn passing through B. The tangent at A meets the second circle at the point C, as shown in the diagram. Write down the coordinates of C.</p> <p>(d) Find the equation of the circle with BC as diameter.</p>	 <p>4</p> <p>1</p>  <p>1</p> <p>2</p>

2

An ear-ring is to be made from silver wire and is designed in the shape of two touching circles with two tangents to the outer circle as shown in Diagram 1.



Diagram 1

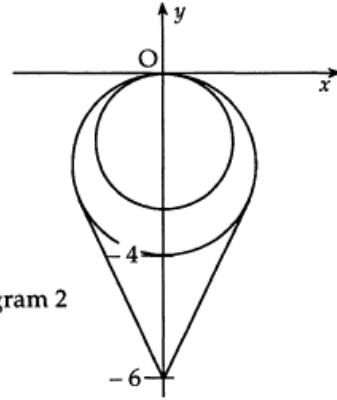


Diagram 2

Diagram 2 shows a drawing of this ear-ring related to the coordinate axes.

The circles touch at $(0, 0)$.

The equation of the inner circle is $x^2 + y^2 + 3y = 0$.

The outer circle intersects the y -axis at $(0, -4)$.

The tangents meet the y -axis at $(0, -6)$.

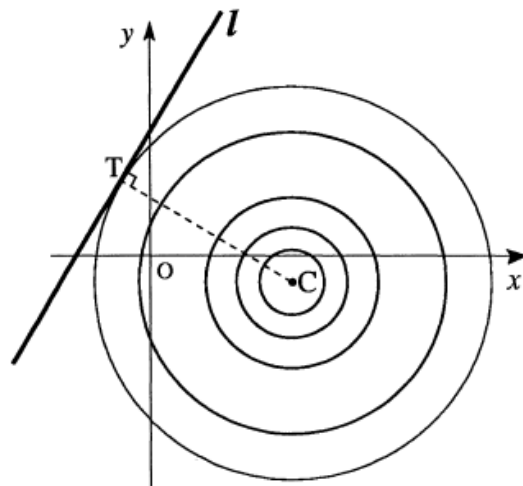
Find the total length of silver wire required to make this ear-ring.

(6)

3

In an experiment with a ripple tank, a series of concentric circles with centre $C(4, -1)$ is formed as shown in the diagram.

The line l with equation $y = 2x + 6$ represents a barrier placed in the tank. The largest complete circle touches the barrier at the point T .



(a) Find the equation of the radius CT .

(3)

(b) Find the equation of the largest complete circle.

(5)

Diagram 1 shows :

- the point $A(1, 2)$,
- the straight line l passing through the origin O and the point A .
- the parabola p with a minimum turning point at O and passing through A .
- and the circle c , centre O , passing through A .

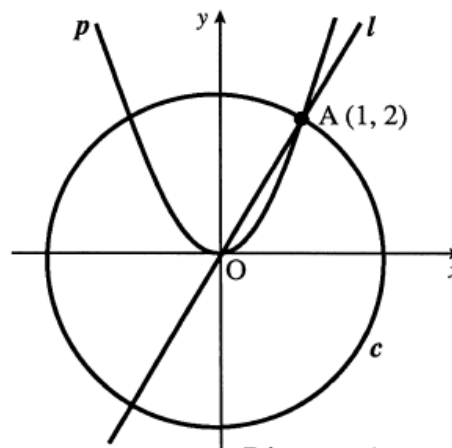


Diagram 1

(a) Write down the equations of the line, the parabola and the circle.

(3)

The following transformations are carried out:

- the line is given a translation of 4 units down (i.e. -4 units in the direction of the y -axis). Diagram 2 shows the line l' , the image of line l , after this translation.
- the parabola is reflected in the x -axis.
- the circle is given a translation of 2 units to the right (i.e. $+2$ units in the direction of the x -axis).

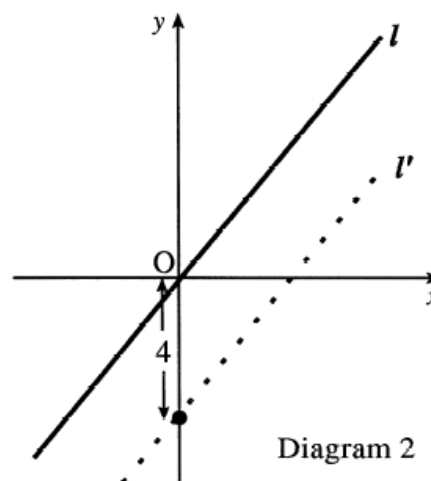


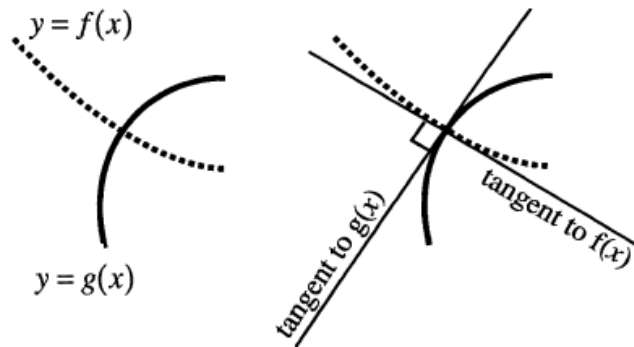
Diagram 2

(b) Write down the equations of l' , p' (the image of the parabola p) and c' (the image of the circle c).

(4)

- (c) (i) Show that the line l' passes through the centre of the circle c' . (1)
- (ii) Find the coordinates of the points where the line l' intersects the parabola p' . (3)

Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



- (a) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y - 5)^2 = 13$. These two curves intersect at $(3, 7)$ and $(-3, 7)$. Prove that these curves are orthogonal.

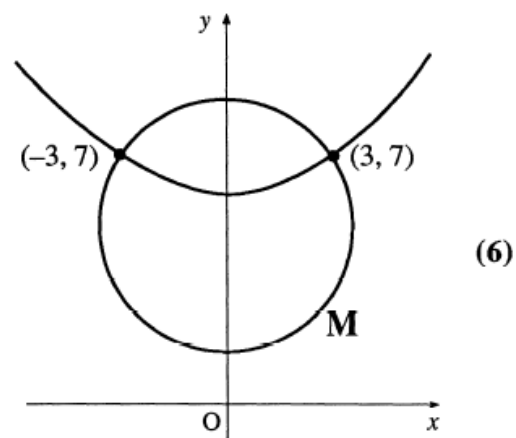


Diagram 1

- (b) Diagram 2 shows the circle M, from (a) above, which is orthogonal to the circle N. The circles intersect at $(3, 7)$ and $(-3, 7)$.
- Write down the equation of the tangent to circle M at the point $(-3, 7)$.
 - Hence find the equation of circle N.

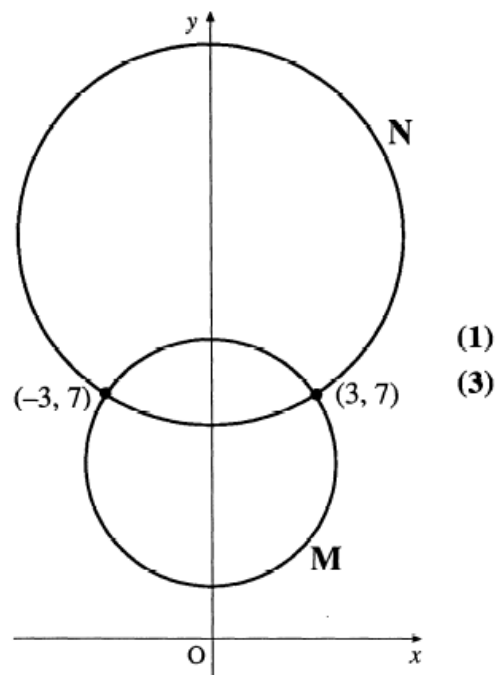


Diagram 2